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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

INTERIM PROGRESS REPORT

TO

JET PROPULSION LABORATORY

J. P. L. CONTRACT NUMBER 950897

J. O. NUMBER 6052/60008

This work was performed for the Jet Propulsion Laboratory,
California Institute of Technology, sponsored by the
National Aeronautics and Space Administration under
Contract NAS7-100.

PREPARED BY

AEROFLEX LABORATORIES INCORPORATED
South Service Road
Plainview, Long Island, New York

14 JULY 1966

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SECTION I

This report summarizes the efforts of Aeroflex Laboratories on J. P. L. Contract No. 950897 since the contract revision of 30 October 1965. The contract goal is an investigation leading to the design of an improved type of Jet Vane Actuator. The report includes accumulated data and analysis of a pivot flexure and two possible approaches to their use in a breadboard Jet Vane actuator.

1. Scope of the Contract

The goal of this investigation is to provide a breadboard of a Jet Vane Actuator which can be refined to meet all physical and environmental conditions encountered during an extended space exposure, and provide reliable, accurate positioning of a vane for mid-course correction of a space probe. The Jet Vane actuator will consist of a moving coil DC torquer and two pivots. The torquer input will be a current controlled amplifier such that torque output is directly proportional to current input; the pivots are to provide a fixed center of rotation and a linear torsional spring rate.

2. Phases of the Contract

The investigation is divided into two parts. Phase I is designed of a flexure which combines radial and axial rigidity with a linear ($T=KG$) torsional spring rate which can be refined to meet the environmental and physical requirements of a Jet Vane Actuator. Phase II requires the integration of two flex pivots with a torquer and testing of the unit to demonstrate feasibility of the concept. After such feasibility

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has been demonstrated and a future specific application appears, suitable design data will be available to design and build an operational system.

3. Tri - Flex Pivot Investigation

The TRI-FLEX pivot was chosen for investigation because, during tests performed as part of this program (see Report of Tri-Flex pivots dated 9 April 1965) it was found to combine high radial and axial rigidity with large torsional deflections (in excess of 50°).

The pivot comprises three radial leaf springs, an inner cage, and an outer cage (see figure 1). One end of each spring is rigidly fixed (welded) to each cage such that the spring passes through the center of rotation, through a clearance hole in the inner cage and is then joined to the outer cage. The three springs are at 120° to each other, and are offset axially for clearance.

Just prior to resumption of the contract, it was observed that the ratio of inner cage radius to leaf spring length represented hereafter by the symbol K is a prime parameter. When the contract was resumed a large Tri-Flex pivot model which permitted varying the spring length for a fixed inner cage radius was constructed. Tests were performed on this model to determine the optimum K ratio. The tests provided data of pivot torsional linearity and sensitivity to radial load. Graphs of torque versus deflection for various K values at 10 lbs., and 17 lbs. radial load and without radial load are included as figures 2 to 8. The data accumulated showed a discontinuity in the torsional linearity at flex.

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pivot free position. The large angular travel ($\pm 60^\circ$) permitted preloading the pivots relative to each other by twice the required travel (45° for $+22\frac{1}{2}^\circ$ travel required) to remove the discontinuity from the operating range. A graph of the results of this test for $K = .111$ appears as figure 9, and for $K = .125$ as figure 10.

While these tests were in progress, an analysis was performed to permit design of Tri-Flex pivots. The analysis along with sample calculations are included as section II of this report. The analysis indicated that a pair of flex pivots which would deflect 22° with a torque of 3 oz. in. and not exceed 2 inches in diameter, using material capable of 90,000 P.S.I. stresses would be 2.5 inches long each.

4. TRI - FLEX Pivot with Tapered Width Springs

The problem of maintaining strength and increasing flexibility of the leaf springs suggested use of a constant stress spring. The Tri - Flex pivot analysis showed that the stress distribution in the leaf springs, due to bending moments, vary from maximum at one end to zero approximately $1/4$ of the spring length from the larger cage to a negative bending moment approximately $1/3$ of the maximum at the large cage. Since the leaf springs could not be made with a zero dimension, a compromise was made. The springs were made of constant width from the outer cage end up to where the stresses returned to the same numerical value. From this point inward, the springs were tapered (outward) to achieve constant stress. This configuration was analyzed and copies of this analysis were sent to J. P. L as part of Progress Report

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Number 19 for the period ending 15 May 1968. A copy is included as section III of this report.

Springs tapered to the proportion described above were tested for linearity and sensitivity to radial load. The results of these test figures 11 through 14 demonstrated the high sensitivity of this spring configuration to radial load and eliminated this as a possible solution to the Jet Vane Actuator flexure problem.

The desirability of a size reduction beyond the constant width flex pivot, and the time required for analysis and accumulation of experimental data had not been anticipated in the contract. This study arose as a result of the previous investigation, and was performed with the concurrence of cognizant J. P. L. engineering. Unfortunately the supplementary study did not produce a useable flexure.

5. Proposals for Continuation of Jet Vane Actuator Investigation

Tests indicate that the torsional spring rate of Tri - Flex pivots at their free position is somewhat lower than that of the remainder of their travel. To avoid this region, it was decided that the pivots be pre-loaded against each other in a Jet Vane Actuator such that the null of a pair of pivots was 22° from the null of each of the individual pivots. This, then, would provide torsional linearity over the entire operating range without any discontinuity. Equations derived for the Tri-Flex pivot show that for other parameters established for the Jet Vane Actuator (natural frequency, angular travel, peak torque) limiting material stresses result in Tri - Flex

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pivots that are too long (2.5 inches long each) or too large in diameter to be practical (see sample calculation # 2, in section II).

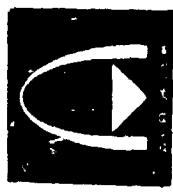
One possible solution would be to use only a small pivot preload which would result in smooth linear response over most of the travel range, particularly at null. It is expected that close manufacturing controls will reduce the torsional discontinuity at null to one degree or less, and satisfactory results will be obtained.

A second approach would reduce the natural frequency requirement approximately 25% with a corresponding reduction in peak torque. This would permit use of a pre-loaded pair of pivots in a Jet Vane Actuator. The torque requirement would be reduced so that a torquer rotor of somewhat lower inertia could be used.

Either approach would require design and fabrication of two pairs of pivots bracketing the desired K (R) values. These would be sized for attachment to the moving coil torquer which has been fabricated by Aeroflex Laboratories as part of this program. These pivots would be tested and a report presented to J. P. L. If the results were satisfactory, the marriage of the torquer to the pivots would be completed; test data on the breadboard Jet Vane Actuator would be taken, and a final report, including fundamental design data required for a finished Jet Vane Actuator, would be submitted.

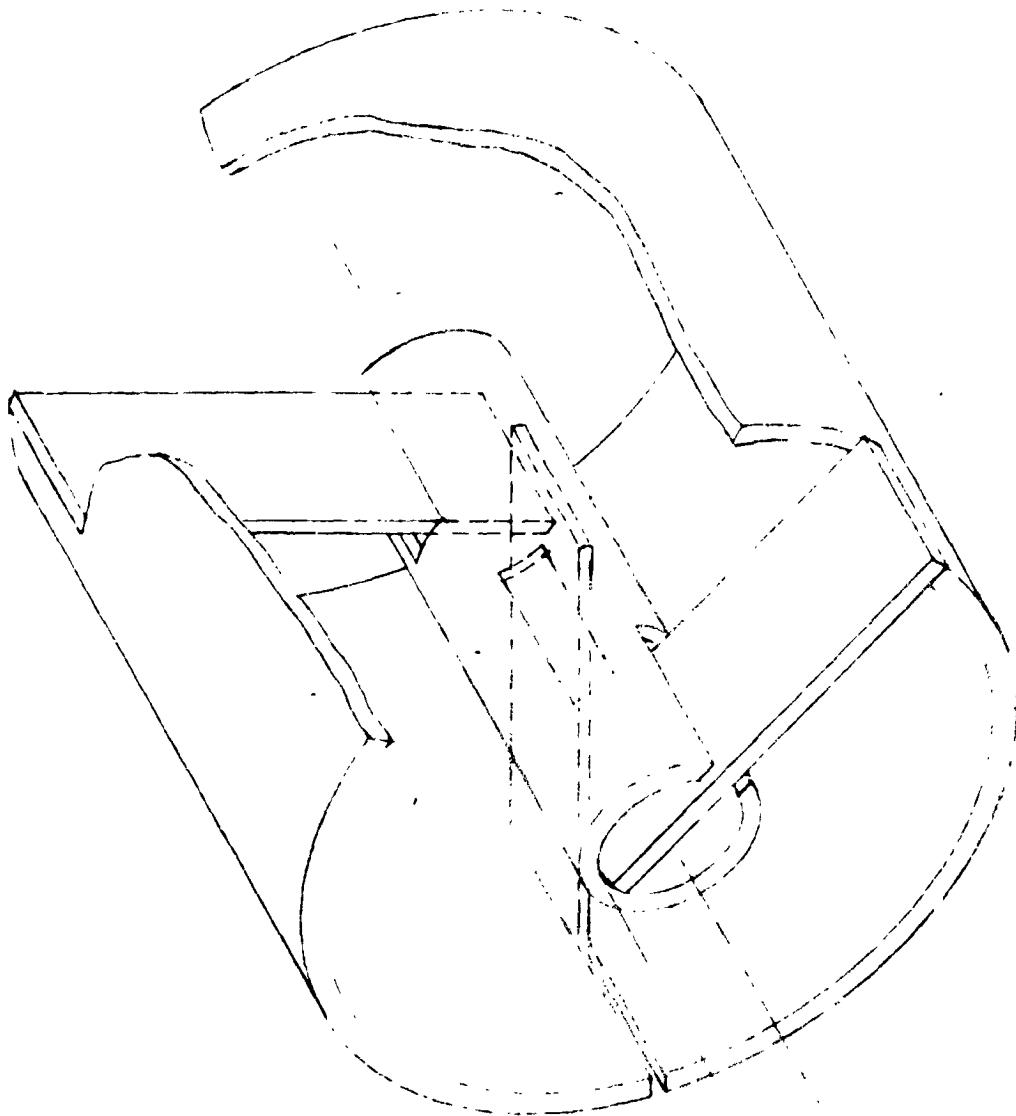
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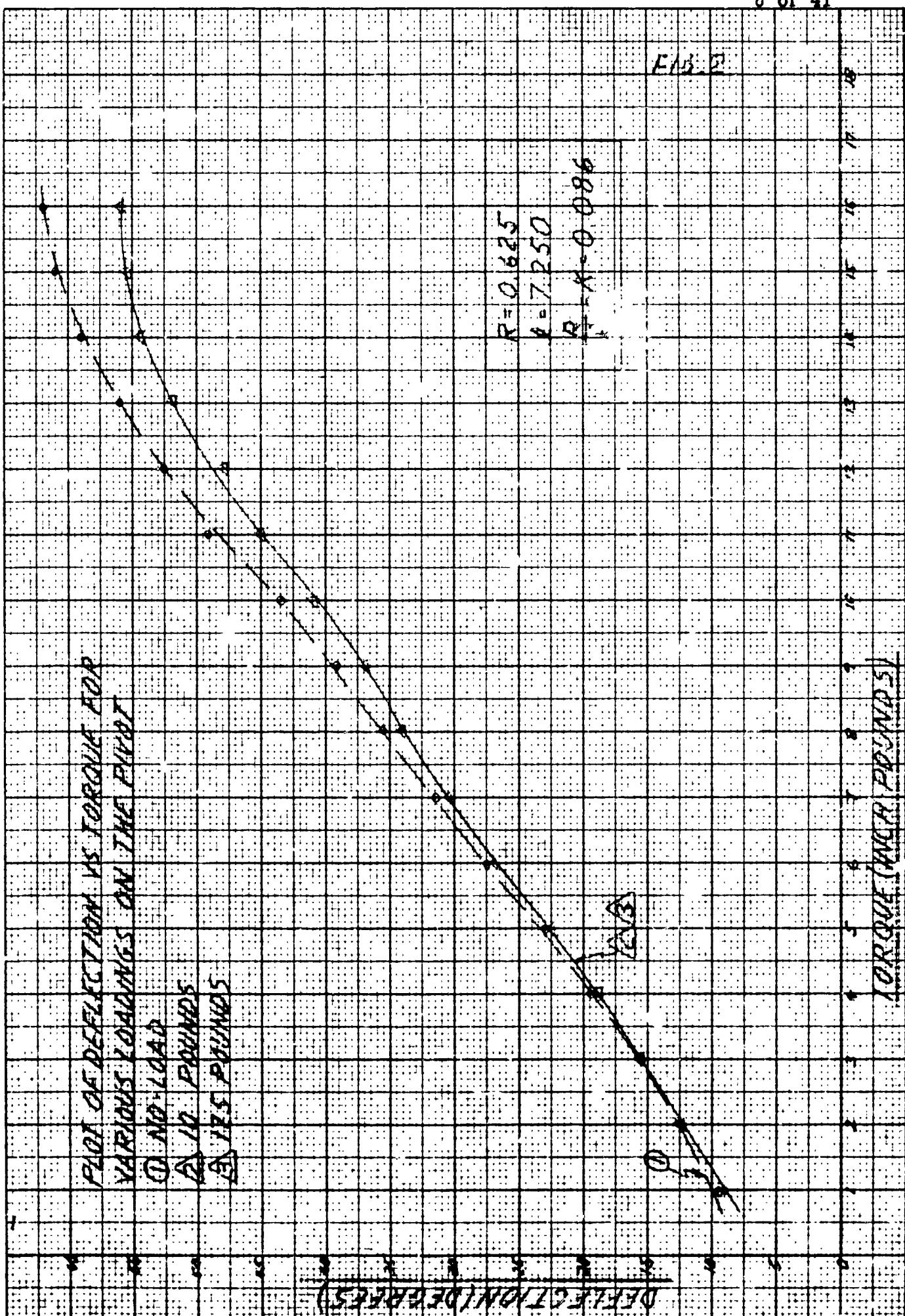
Either approach would require approximately four (4) calander months. Present funding, however, will cover only one approach, and it is desired that J. P. L. direct Aeroflex Laboratories as to the preferred configuration.



AEROFLEX LABORATORIES INC.
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FIG. 1





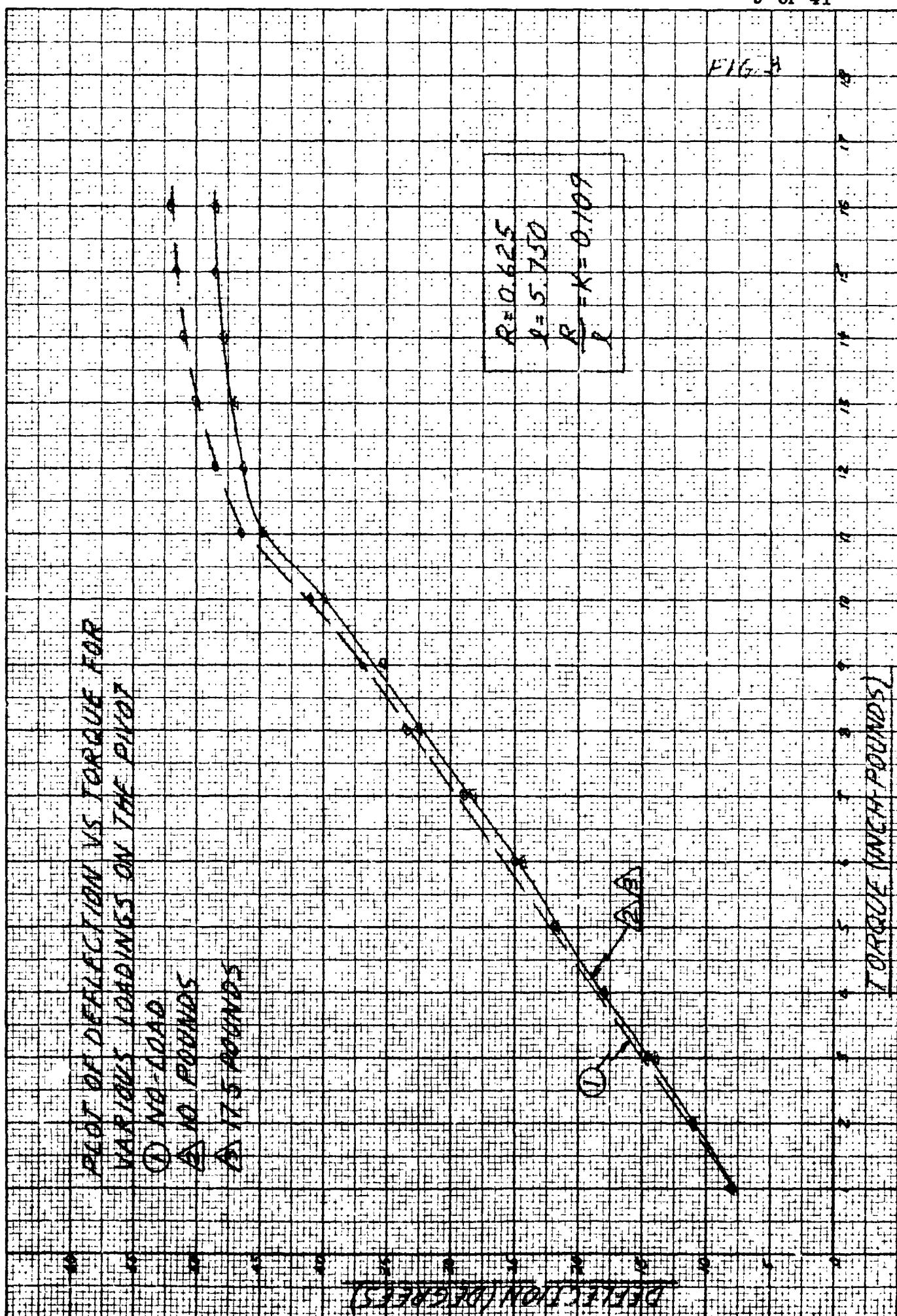


FIG. 8

PLOT OF DEFLECTION VS TORQUE
DEFINITION DUE TO 1/25 POUND
LOAD AT 1/25 INCH THAN .4.

ANGULAR DEFLECTION - DEGREES

$$\begin{aligned} R &= 6.25 \\ L &= 5.625 \\ K &= 11 \end{aligned}$$

18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 TORQUE - POUNDS INCHES

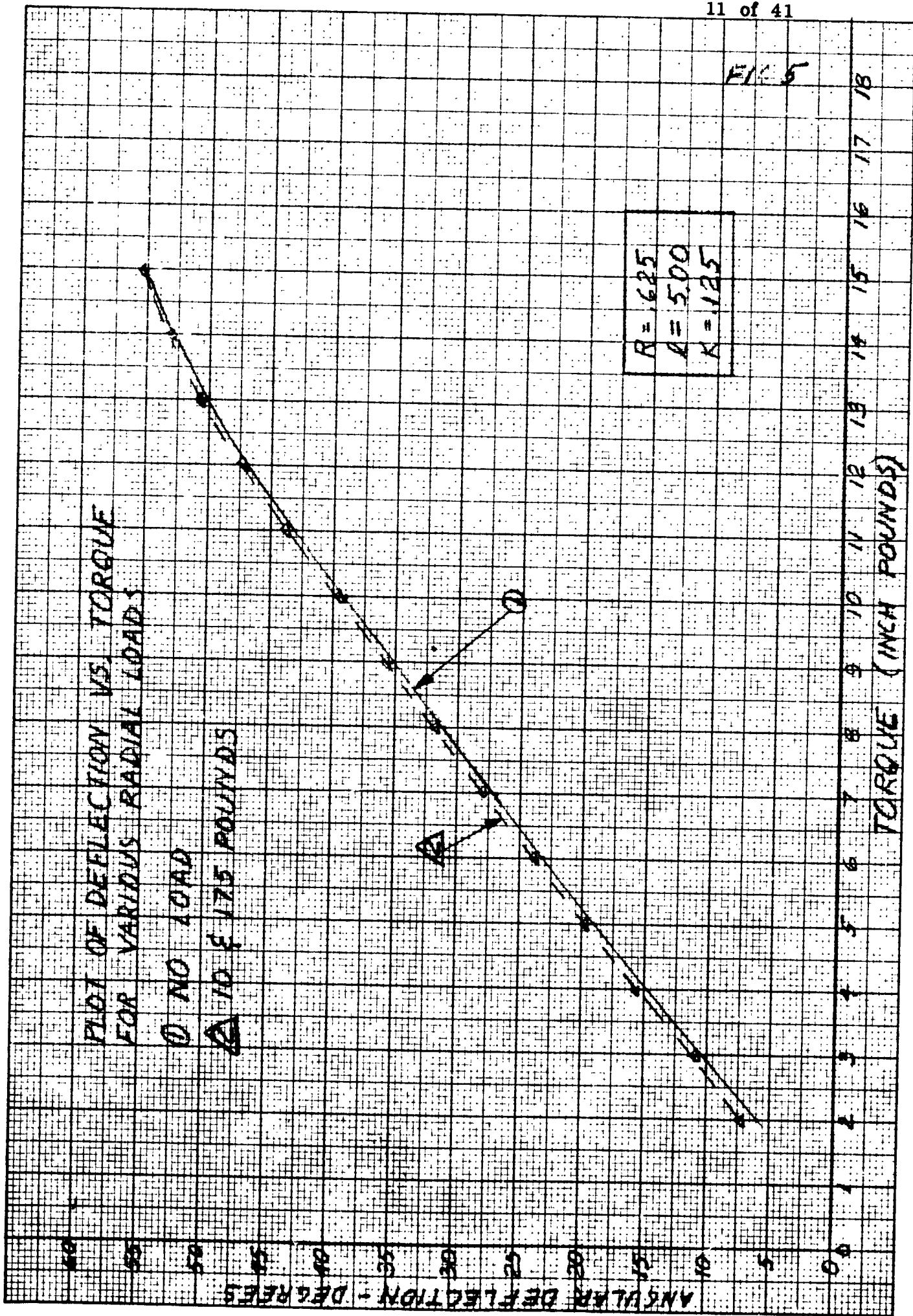


FIG. A

PLOT OF DEFLECTION VS TORQUE FOR
VARIOUS LOADS OF THE PLATE
IN POUNDS

A 15 POUNDS

B 10 POUNDS

C 5 POUNDS

D 2 POUNDS

E 1 POUND

(SHEET 10 OF 11)

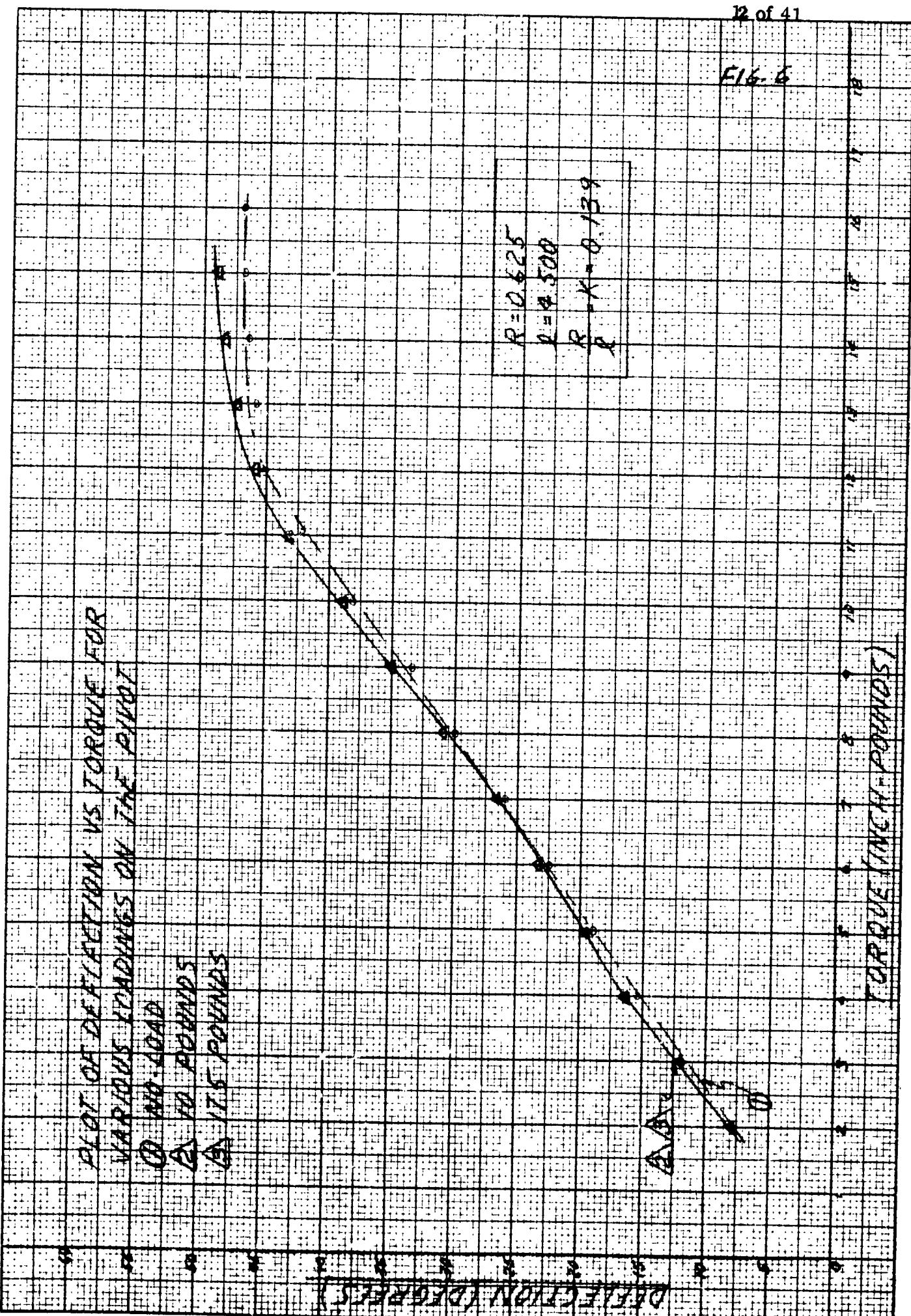
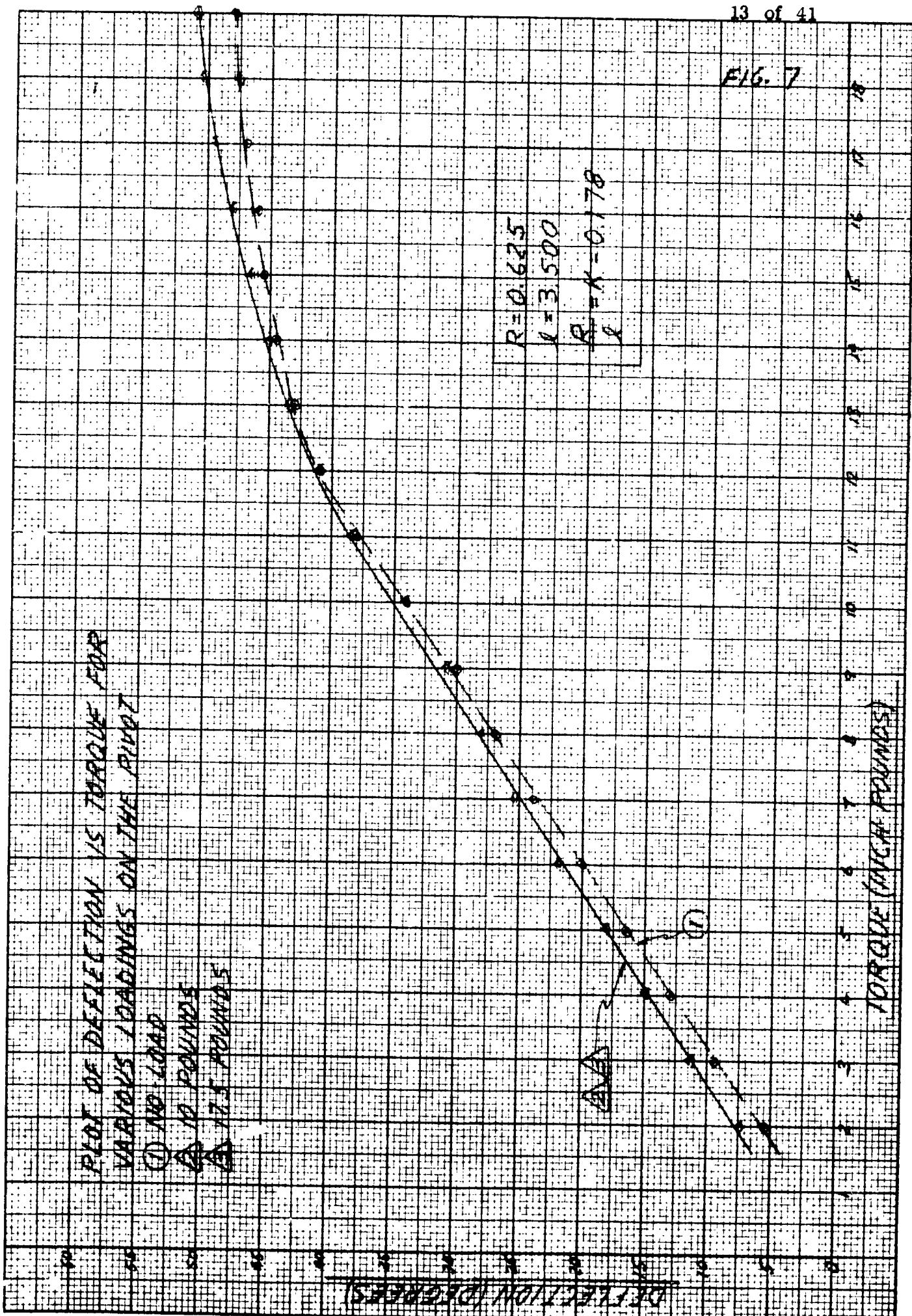
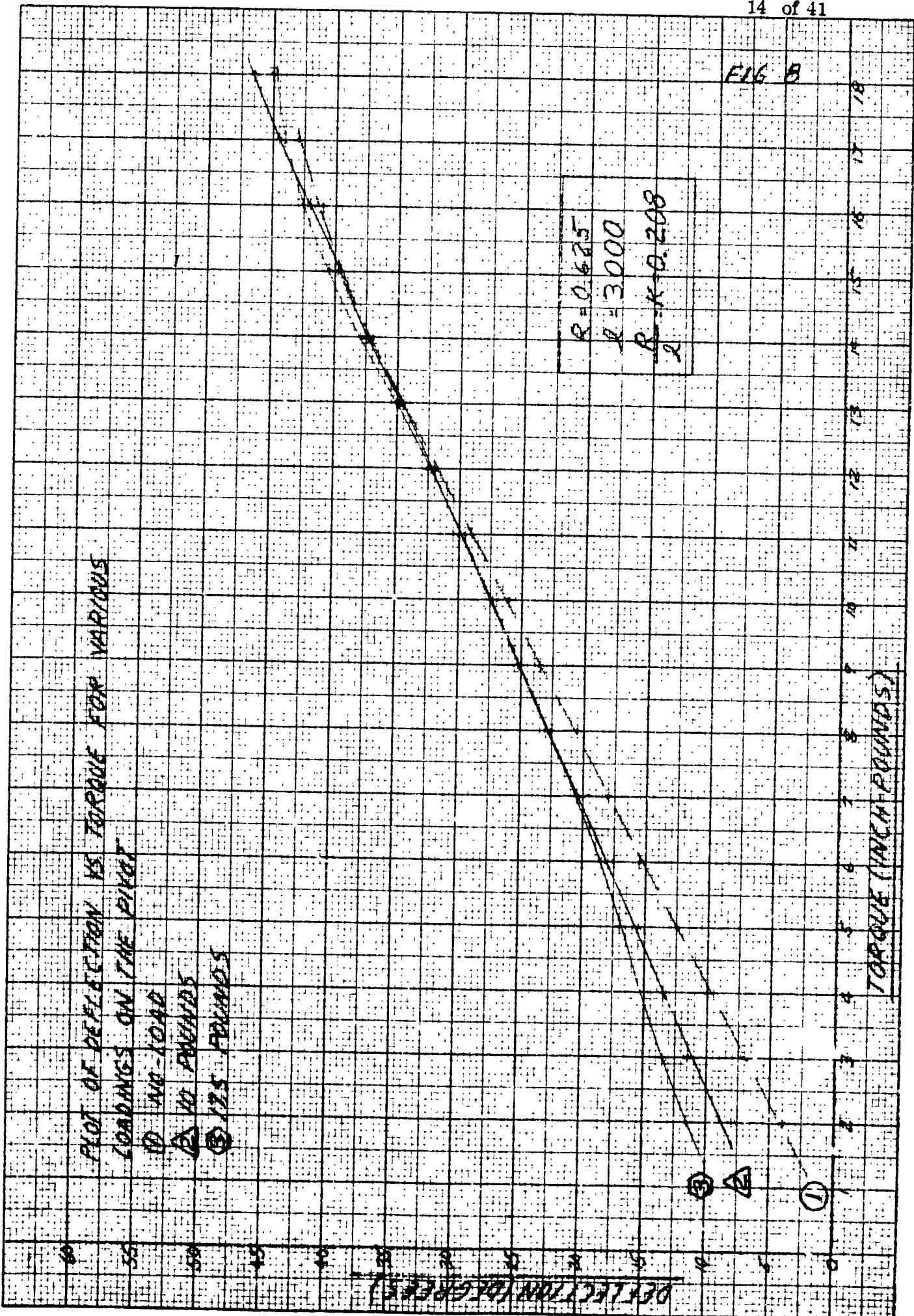


FIG. V



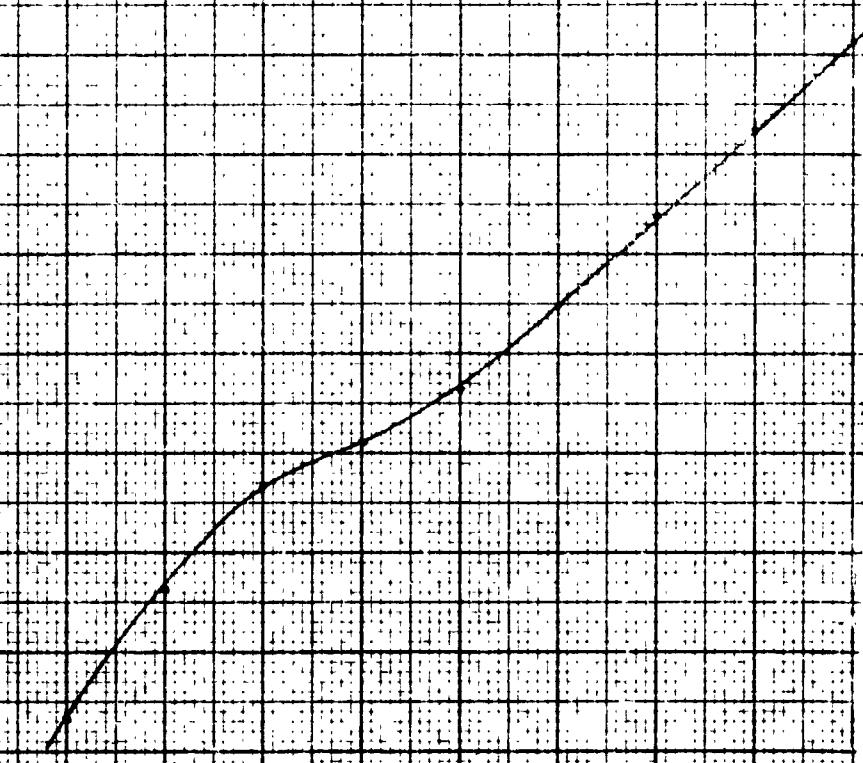


K&E 10 X 10 TO TUFF-L INCH
KELVIN & LESTER CO.

ANGULAR DEFLECTION - DEGREES

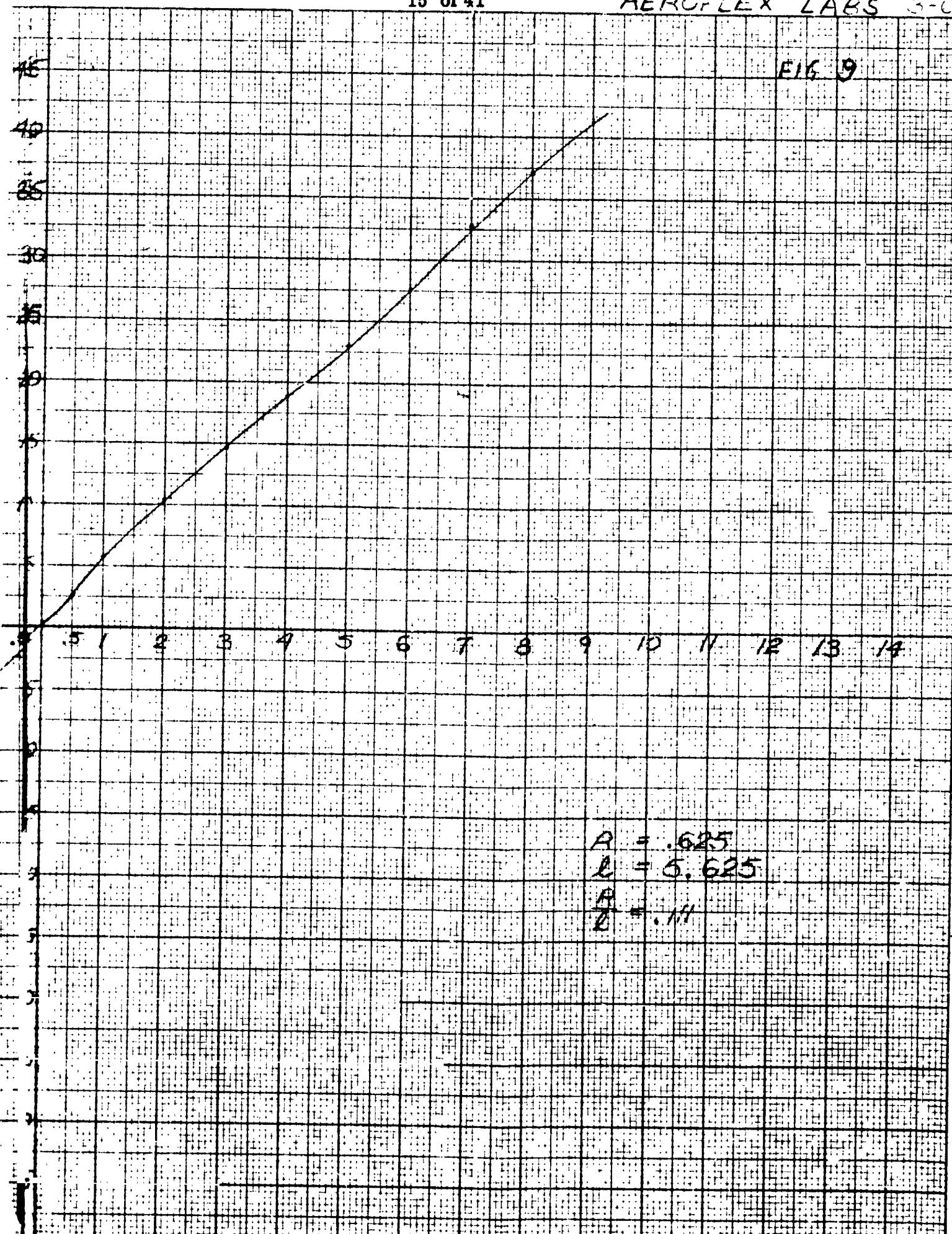
2 SINGLE STAGE
FLEXURES 45°
OPPOSED

14 13 12 11 10 9 8 7 6 5 4 3 2 1



TORQUE

FIG. 9



$$\begin{aligned}P &= .625 \\L &= 5.625\end{aligned}$$

$$R = .11$$

ANGULAR DEFLECTION DEGREES

TWO FLEX PIVOTS
PRELOADED 90°
(FREE POSITION OF EACH
45° FROM PRELOADED
FREE POSITION)

CURVE O NO RADIAL LOAD

CURVE △ 10 POUND RADIAL LOAD

TORQUE - POUND INCHES

14 13 12 11 10 9 8 7 6 5 4 3 2 1

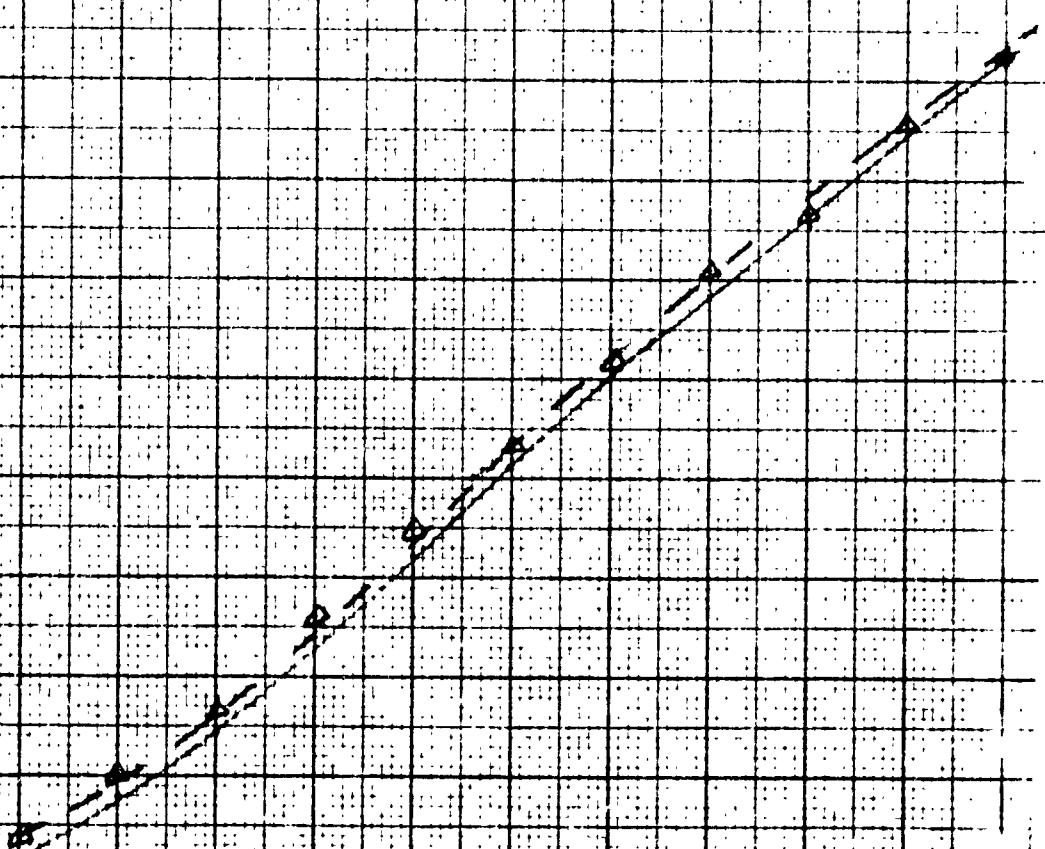


FIG. 10

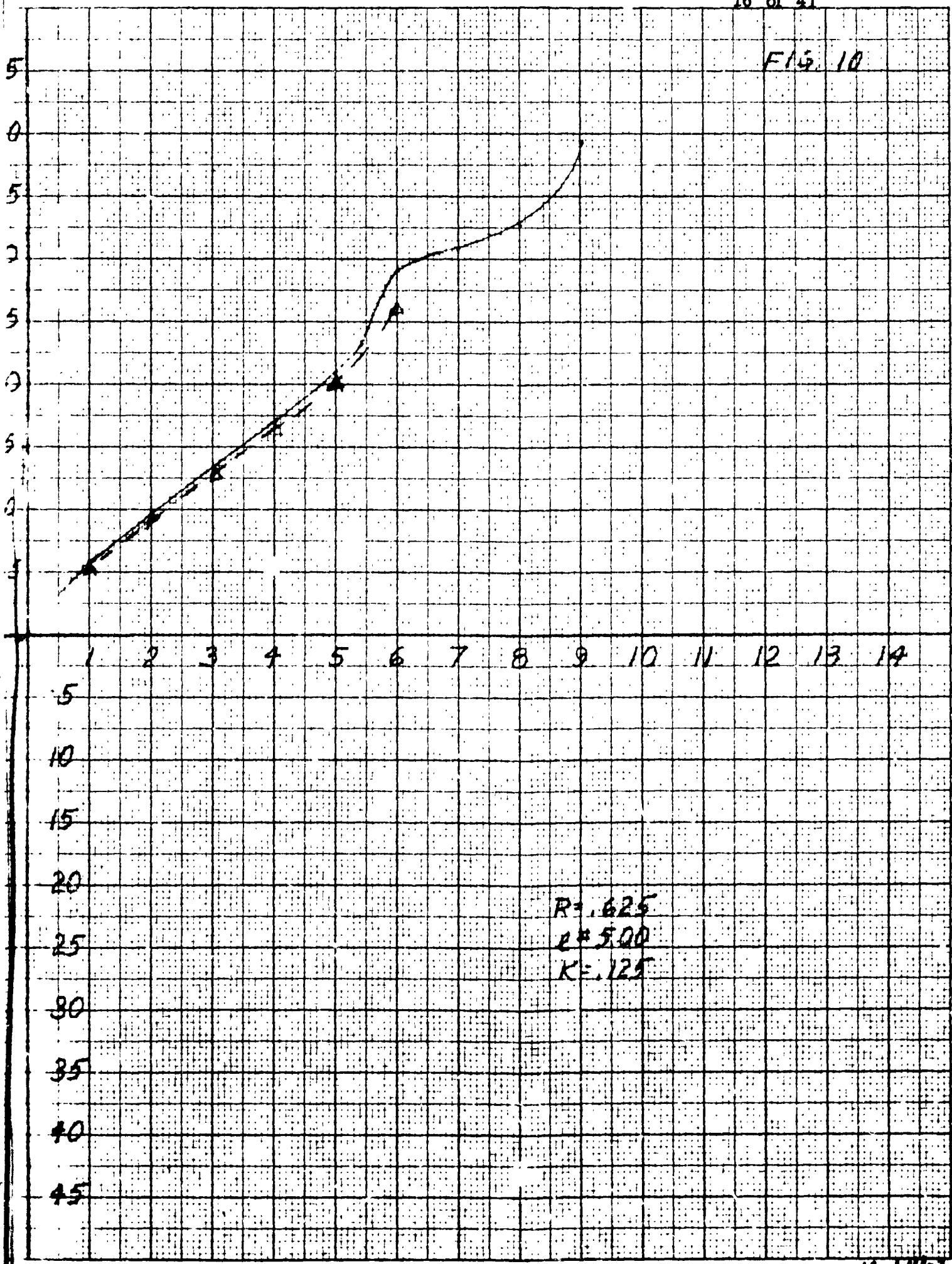


FIG. 11

TAPERED SPRING A

$A = 281$

$K = .096$

$R = 62.5$

$d = .0167$

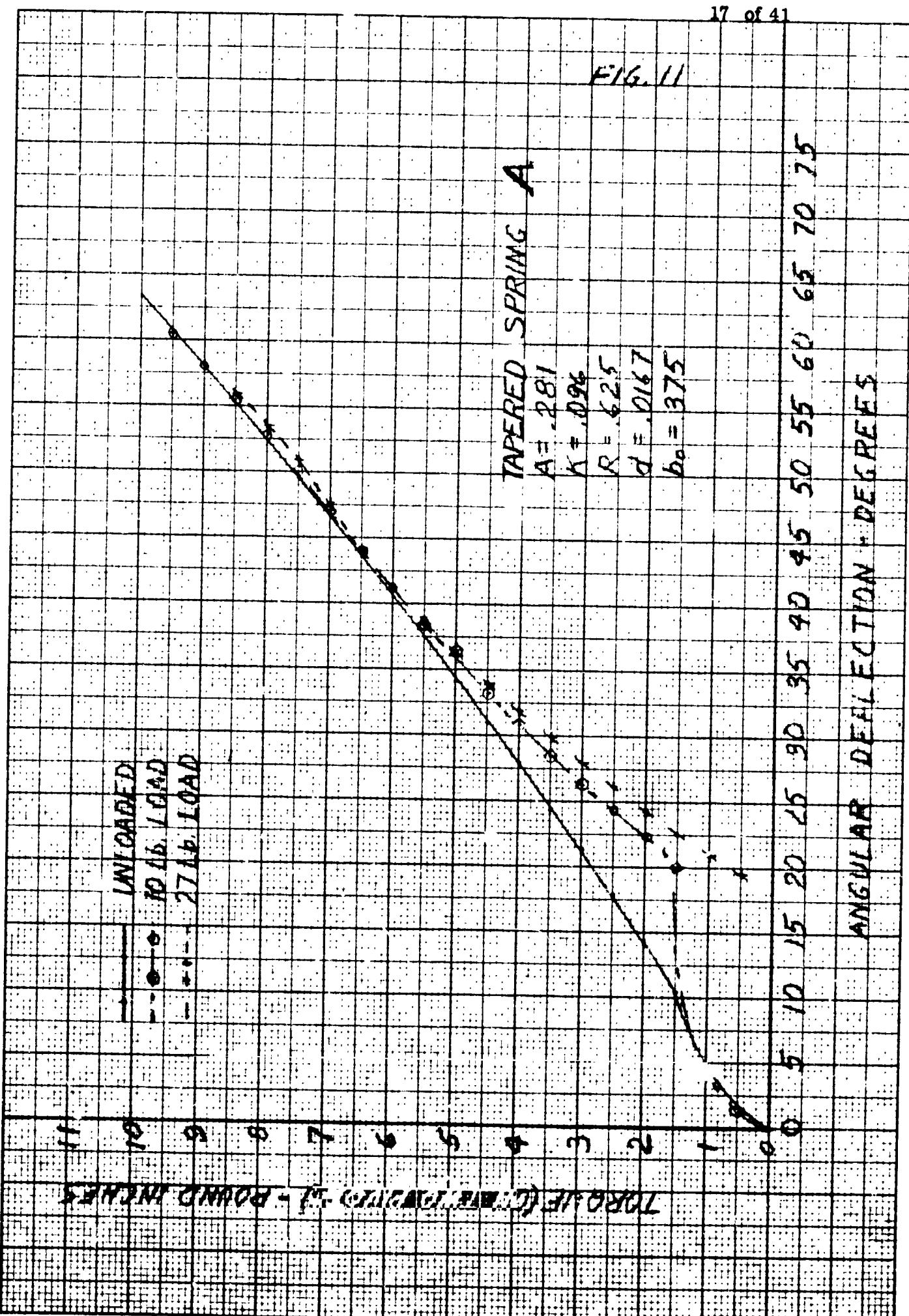
$b_0 = 375$

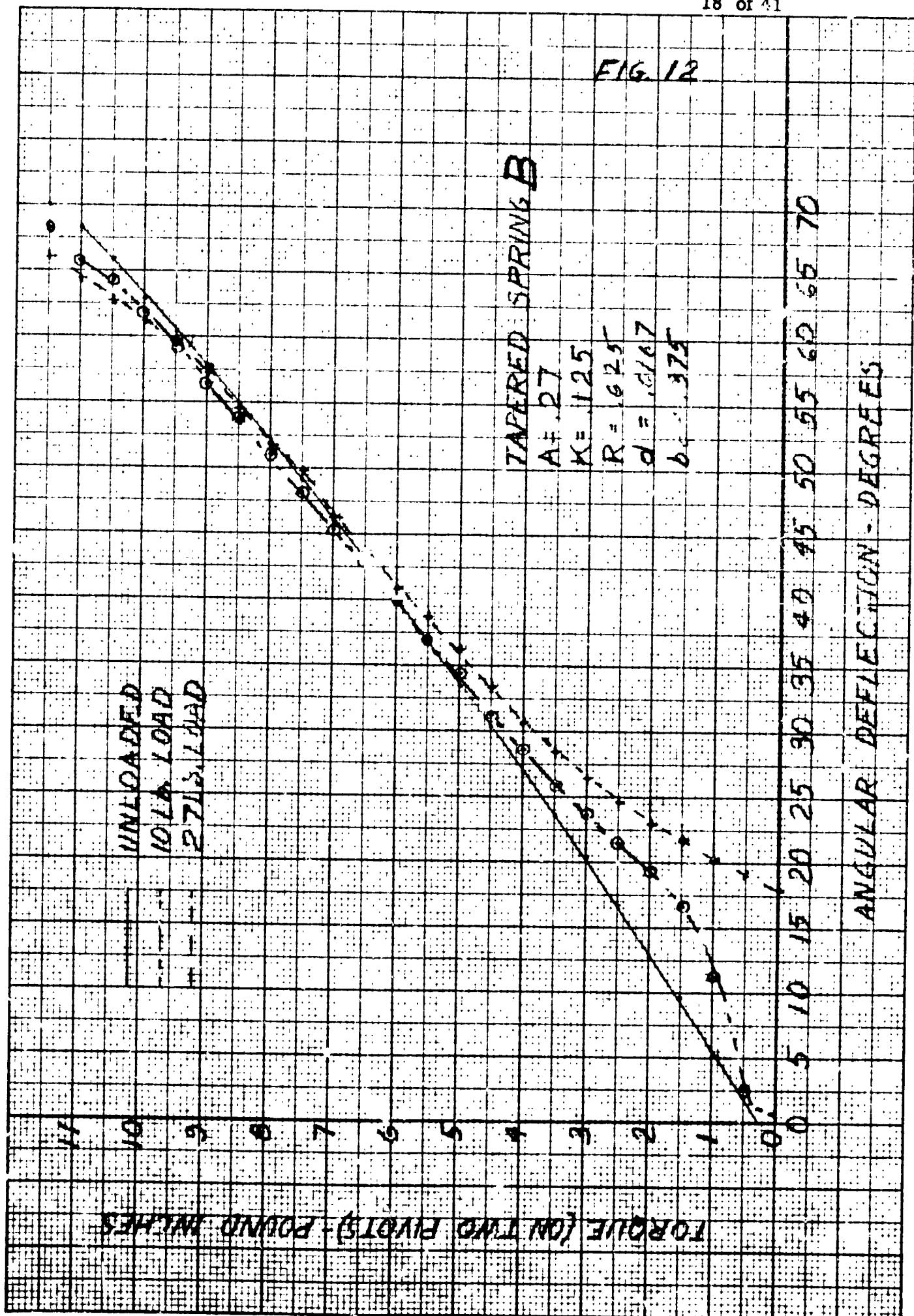
100 90 80 70 60 50 40 30 20 15 10 5 0

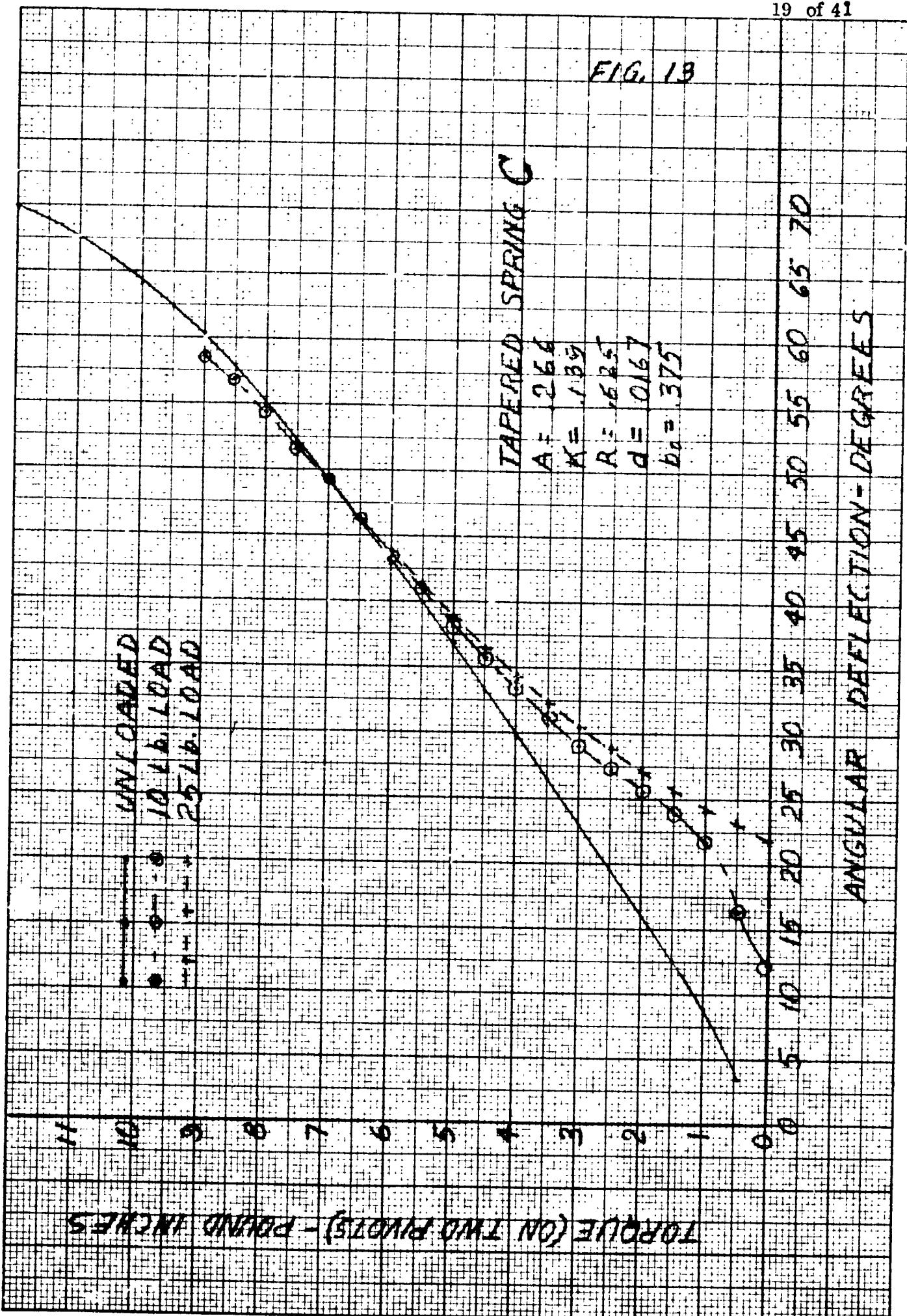
ANGULAR DEFLECTION - DEGREES

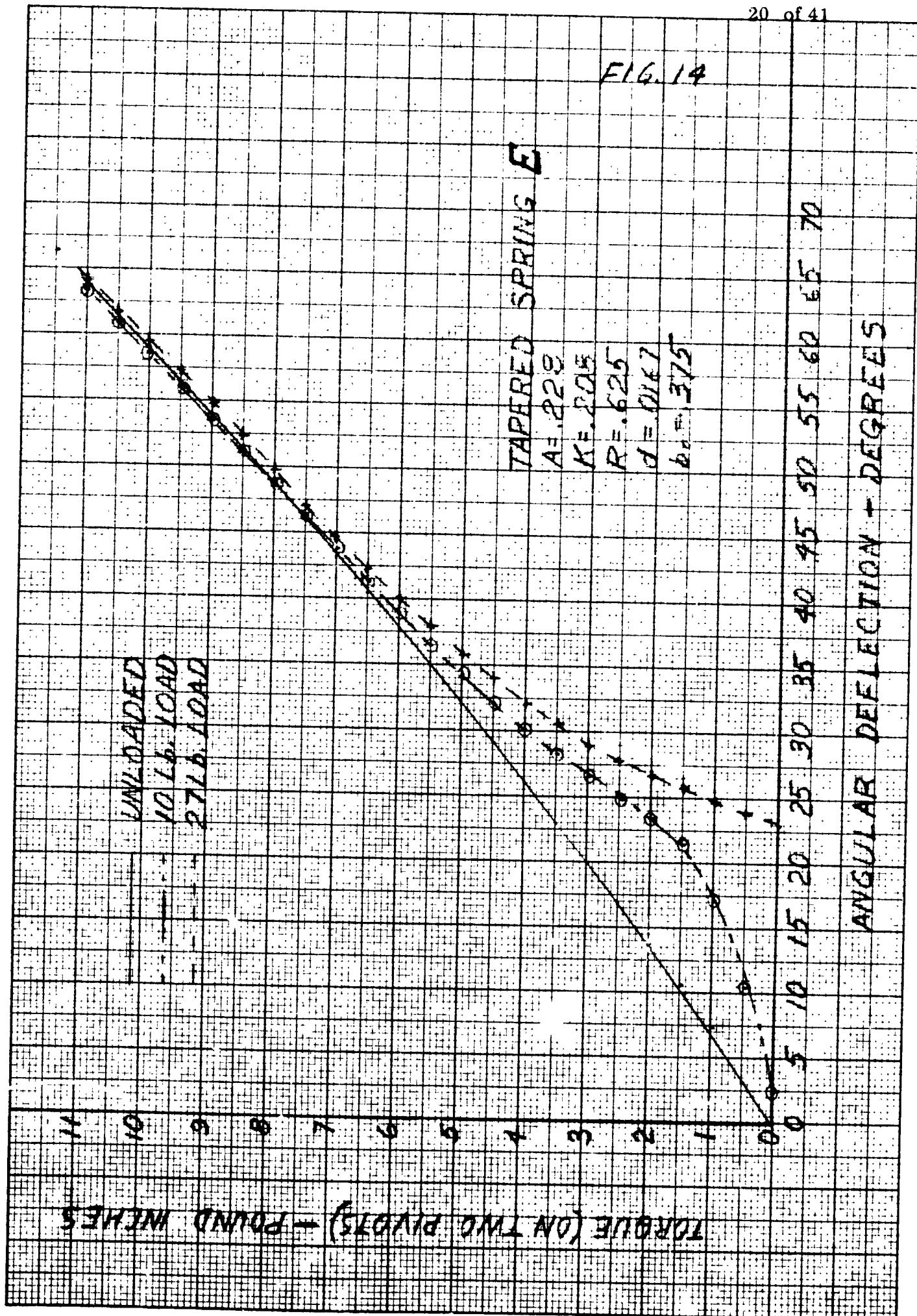
UNLOADED
10 lb LOAD
27 lb LOAD

27 LB LOAD - PT. CIRCUMFERENCE DIA. 370.802









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SECTION II

General Description

The Tri-Flex Pivot is a device which combines radial and axial rigidity with low controlled torsional flexibility. It permits large angular deflections ($\pm 60^\circ$). The torsional spring rate is almost constant over most of this range, and is not appreciably affected by radial load.

The Tri-Flex Pivot comprises three radial lead springs, an inner cage and an outer cage (see figure 1). One end at each leaf spring is rigidly fixed (welded) to each cage such that the leaf passes through the center of rotation, through a clearance hole in the inner cage and then is joined to the outer cage. The three leaves are at 120° with respect to each other and are offset axially for clearance.

Tri-Flex Pivots support radial forces by tensile forces in the leaf springs. Allowable loadings are limited by cross sectional area of a single leaf spring. Axial forces induce bending of the leaf springs across their width. The springs are relatively stiff and strong in this direction, and if lateral buckling is avoided, the pivots may be designed to withstand reasonably high loads. Flex Pivots should be used in pairs to limit bending moments perpendicular to the rotational axis.

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The ratio of the smaller cage inner radius divided by the leaf spring length, represented by the symbol K in calculations included with this report, is a significant parameter. A specific range of values of this parameter provide linearity of the torque deflection curve and insensitivity to radial loads.

Values from .105 to .14 provide linearity within $\pm 2\%$ over 50° of travel with an optimum value of .115 - .12 where linearity within 1% can be obtained for travel of up to 50° .

The derivation and sample calculations included in this report neglect radial pivot forces (axial tension and compression in the leaf springs). This affects the accuracy of spring rate calculations. Figure 15 contains a correction factor for this parameter. Graphs of Spring Rate Coefficient and Stress Factor are also included as figures 16 and 17.

The pivots tested as of the writing of this report have exhibited a slight buckling at their zero position. This buckling appears as a deviation of the Torque Deflection line from zero degrees at the zero torque point. The deviation ranges from $.5^\circ$ to 4.5° . It is not known if this is an inherent characteristic or if this is a result of misalignment which has occurred in the prototype set up. The pivots have been used in preloaded pairs to eliminate this condition at midrange.

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SYMBOLS

θ	=	deflection angle of rotor relative to station radions
P	=	force lbs.
l	=	length of leaf spring inches
M	=	leaf bending mount at inner cage lb. in.
T	=	Torque on Flex Pivot
E	=	Young's Modulus lb/in^2
R	=	inner radius of inner cage inches
K	=	$\frac{R}{l}$
B	=	Spring Rate Coefficient
C	=	Stress Factor

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TRI-FLEX PIVOT ANALYSIS

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FOR A SINGLE LEAF OF TRI-FLEX-PIVOT

$$1. \theta = \frac{PL^2}{2EI} + \frac{M_0 l}{EI}$$

$$2. P = \frac{6EI}{l^2} \left(\frac{M_0 l}{EI} - \theta \right)$$

$$\text{DEFLECTION} = P\theta$$

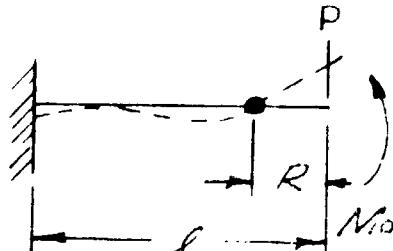


FIGURE 1

$$3. J = R\theta = \frac{-PL^3}{3EI} + \frac{M_0 l^2}{2EI} - \left[\frac{2EI}{l^2} \left(\frac{M_0 l}{EI} - \theta \right) \right] \frac{l^3}{3EI} + \frac{M_0 l}{2EI}$$

$$R\theta = \frac{2l}{3} \left(\frac{M_0 l}{EI} - \theta \right) + \frac{M_0 l^2}{2EI} = \frac{-M_0 l^2}{6EI} + \frac{2l}{3} \theta$$

$$4. M_0/\text{LEAF} = \left(\frac{6EI}{l^2} \right) \theta \left(\frac{2l}{3} - \theta \right) = \frac{2EI\theta}{l^2} (2l - 3\theta)$$

$$\text{FROM 2. } P = \frac{2EI}{l^2} \left\{ \left[\frac{2EI\theta}{l^2} (2l - 3\theta) \right] \frac{l}{EI} - \theta \right\} = \frac{2EI}{l^2} \left\{ \left(\frac{4l - 6\theta}{l} \right) \theta - \theta^2 \right\}$$

$$5. P = \frac{6EI\theta}{l^3} (l - 2R)$$

ON TRI-FLEX-PIVOT

TORQUE $T = I(M_0 - P.R)$

EXAMPLE 4.4.5.

$$= 3 \left\{ \frac{2EI\theta}{l^2} (l - 3R) - \frac{6EI\theta}{l^3} (l - 2R, R) \right\}$$

OUTER CAGE
INNER CAGE

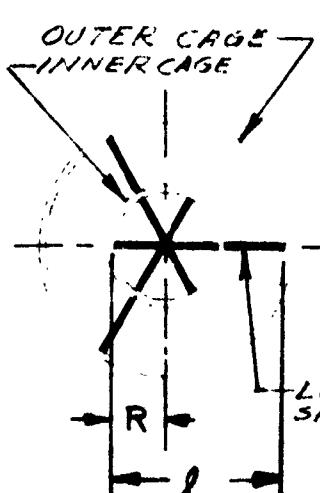
$$\frac{T}{\theta} = \frac{3EI}{l^2} \left(\frac{4l^2 - 6lR - 6lR^2 + 12R^3}{l} \right)$$

$$\frac{T}{\theta} = \frac{12EI}{l^3} (l^2 - 3lR + 3R^2)$$

$$\frac{T}{M} = 3 \left(1 - \frac{PR}{M_0} \right) = 3 \left\{ 1 - \left[\frac{\frac{6EI\theta}{l^3} (l - 2R) R}{\frac{2EI\theta}{l^2} (2l - 3R)} \right] \right\}$$

$$= 3 \left(1 - \frac{3(l - 2R) R}{l(2l - 3R)} \right) = 3 \left(\frac{2l^2 - 3lR - 3lR + 6R^2}{l(2l - 3R)} \right)$$

$$= 6 \left(\frac{1 - 3lR + 3R^2}{l(2l - 3R)} \right)$$



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$$\text{SETTING } K = \frac{R}{\ell} \quad \ell = \frac{R}{K}$$

$$\frac{T}{\theta} = \frac{12EI}{\ell^3} (2^2 - 3K^2 + 3R^2)$$

$$= \frac{12EI}{\ell} (1 - 3K + 3K^2) = \frac{12EI}{R} K (1 - 3K + K^2)$$

$$\frac{T}{\theta} = \frac{EI}{R} (12K)(1 - 3K + 3K^2) = \frac{EI}{R} B$$

WHERE B IS THE SWING RATE COEFFICIENT
MAX MOMENT IS AT ROTOR (SMALLER) RADIUS

$$\frac{T}{M} = 6 \left(\frac{L^2 - 3LP + 3R^2}{C(2L - 3K)} \right)$$

$$= \frac{6(1 - 3K + 3K^2)}{(2K - 3K)} = 2(1 - 3K) + \frac{C}{2 - 3K}$$

$$\frac{T}{M_0} = 2(1 - 3K) + \frac{2}{2 - 3K} = C$$

WHERE C IS THE STRESS FACTOR

MAX MOMENT

$$M_0 = \frac{2EI\theta}{\ell^2} (2L - 3R)$$

$$= \frac{EI\theta}{\ell} (2 - 3K) =$$

$$M_{\max} = \frac{EI\theta}{R} K(2 - 3K) \quad \text{WHERE } S = \frac{MC}{I}, C = \frac{d}{2}, I = \frac{bd^3}{12}$$

$$\frac{M}{I} : \frac{S}{C} = \frac{2S}{d}$$

$$\theta = \frac{R}{E} \left(\frac{M_0}{I} \right) \frac{1}{K(2-3K)} = \frac{R}{E} \left(\frac{2S}{d} \right) \frac{1}{K(2-3K)}$$

$$\theta = \frac{RS}{Ed} \frac{2}{K(2-3K)}$$

FIG. 15

**SPRING RATE
CORRECTION FACTOR**

1.50

1.40

1.30

1.20

1.10

1.00

90

80

0 02 04 06 08 10 12 14 15 16 18 20 22

$$K = \frac{S}{P}$$

FIG. 16

CONSTANT WIDTH SPRINGS

$$B = \frac{T}{EI} = 12K(1 - 3K + 3K^2)$$

1.4

1.3

1.2

1.1

1.0

.9

B

.8

.7

.6

.5

.4

.3

.2

.1

.0

0 .02 .04 .06 .08 10 12 14 .16 18 20 22

$$H = \frac{R}{\theta}$$

SPRING RATE COEFFICIENT

FIG. 17

27

26

 $\frac{I}{M_0}$

25

STRESS FACTOR C

24

$$C = \frac{T}{M_0} = 26.3K + \frac{2}{2-3K}$$

23

T = TORQUE (6.6 IN)

M_0 = MAX LEAF SPRING
BENDING MOMENT (6.1 IN)

22

21

20

0 02 04 06 08 10 12 14 16 18 20 22

$$K = \frac{P}{L}$$

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CALCULATIONSJOB 60018

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CONSTANT I. LTH

A. WITHOUT PRELOAD

FOR 3.02 IN T 22° 0.117" 0.117" 0.117"

$$T = 1.5 \text{ oz in/pivot} \cdot \frac{1.5}{.6} = .094 \text{ # in/in.}^2$$

$$\theta = 22^\circ \cdot \frac{\pi}{180} \text{ rad} = .374 \text{ rad}$$

$$\frac{T}{\theta} = \frac{.094 \text{ # in}}{.374} = .251 \text{ # in/in.}$$

U.S. AG R = .12" (FROM GRAPH)

$$A = \frac{T}{E\theta} = \frac{.251}{14.8 \times .374} = .187 \text{ in}^2 \quad \text{if } 10 \cdot .187 = 1.87 \text{ in}$$

$$2(EA) = 1.87 \text{ in} = 2 \left(\frac{F}{R} - R \right) \cdot 2 \left(\frac{.251}{.12} \text{ in} \right) = 14.83 R$$

$$R = \frac{1.87}{14.83} = .1263 \text{ in}$$

$$E = 30 \times 10^6$$

$$I = \frac{T}{EA} = \frac{.251}{\frac{30 \times 10^6 \times .187}{.1263}} = 1.072 \times 10^{-9} \text{ in}^4$$

$$B = \frac{T}{M_0} = 2.497 \quad (\text{FROM GRAPH})$$

$$M_0 = \frac{T}{B} = \frac{.094}{2.497} = .0377 \text{ in}$$

$$S_{MAX} = 90,000 \text{ PSI} \quad S = \frac{MC}{I}$$

$$C = \frac{SI}{M} = \frac{90000 \times 1.072 \times 10^{-9}}{.0377} = .00256 \text{ in}$$

$$\text{WIDTH } b = \frac{12I}{d^3} \text{ WHERE } d = 2C = \frac{12 \times 1.072 \times 10^{-9}}{(0.00512)^3} = .0953 \text{ in}$$

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DATE 8 July 66TRI-FLEX PIVOT SAMPLE
CALCULATIONSJOB 60005

REPORT _____

CONSTANT WIDTH STRUTS
WITH PRELOAD

3 OZ IN

 $\theta = 22^\circ$

0.172"

2 KIPS

$$\theta = 2 \times 22^\circ = 2 \left(\frac{\pi}{180} \cdot \pi \right) = .748 \text{ RAD}$$

$$T = 3.0 \text{ OZ IN ON ONE PIVOT} = \frac{3.0}{16} = .185 \text{ #}"$$

$$\frac{T}{\theta} = \frac{.185}{.748} = .251 \text{ #}/\text{RAD}$$

USING $K = .12$ (FROM STRESS)

$$A = \frac{I}{EI} = .787 \quad \text{FOR } 1.875 \text{ I.D.}$$

$$2(L-R) = 1.875 = 2 \left(\frac{R}{.12} - R \right) = 2 \left(\frac{.0833}{.12} \right) = 14.83 \text{ FT}$$

$$R = \frac{14.75}{14.83} = .1263$$

$$\text{FOR } E = 30 \times 10^6 \quad T = \frac{\theta}{EA} = \frac{.251}{\frac{30 \times 10^6 \times .787}{.1263}} = 1.072 \times 10^{-9}$$

$$B = \frac{T}{M_0} = 2.447 \quad (\text{FROM GEOMETRY})$$

$$M_0 = \frac{T}{B} = \frac{.188}{2.447} = .0754 \text{ #}"$$

$$S_{MAX} = 90,000 \text{ PSI} \quad S = \frac{MC}{I}$$

$$C = \frac{S.I}{M} = \frac{90,000 \times 1.072 \times 10^{-9}}{.0754} = .00128"$$

$$\text{WIDTH } b = \frac{12I}{\sigma^3} \quad \text{WHERE } \sigma = 2C$$

$$= \frac{12 \times 1.072 \times 10^{-9}}{2(.00128)} = .7664/\text{LEAF}$$

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J. P. L. CONTRACT NO. 950897

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SECTION III

1. Analysis of Tri-Flex Pivot with Tapered Width Springs

The analysis, of a Tri-Flex pivot with tapered width springs, presented here, neglects radial pivot (axial leaf spring) forces. These forces were neglected because it was expected that experimentation would permit determination of a specific $A \left(\frac{M^2}{P_e} \right)$ or $K \left(\frac{R}{\delta} \right)$ ratio which would be insensitive to radial pivot loads. This ratio was not found in the course of experimentation, but the analysis and graphs of configuration (figure 18) and Spring Rate Coefficient (fig. 19) are submitted here with an appendix which contains the derivation for angular and lateral deflection of a tapered beam with an end moment.

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TRI FLEX PIVOT ANALYSIS
WITH VARIABLE WIDTH SPRINGS

JOB 60008

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FOR A CANTILEVER BEAM END LOADED AS SHOWN IN FIG 1, THE BENDING MOMENT DIAGRAM TAKES THE FORM SHOWN IN FIG 2. NOTE THAT M_2 IS TAKEN AS A NEGATIVE VALUE.

THEN

$$\frac{x}{l} = -\frac{M_2}{Pl}$$

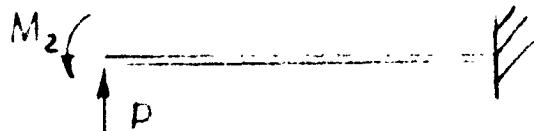


FIG 1

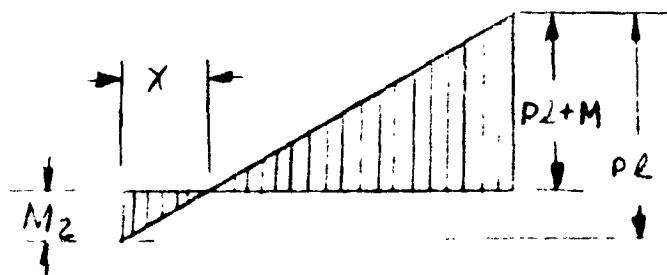


FIG 2

SINCE A CONSTANT STRESS BEAM FOR THIS LOADING CONDITION WOULD APPROACH ZERO WIDTH AT X, A MODIFIED CONFIGURATION SHOWN IN FIG. 3 WILL BE USED WHERE THE BENDING MOMENT AT SECTION 1 IS $-M_2$. THEN

$$l_{12} = 2x = -\frac{2M_2}{Pl}l = -\frac{2M_2}{P}$$

$$l_{01} = l - \left(-\frac{2M_2}{P}\right) = \frac{Pl + 2M_2}{P}$$

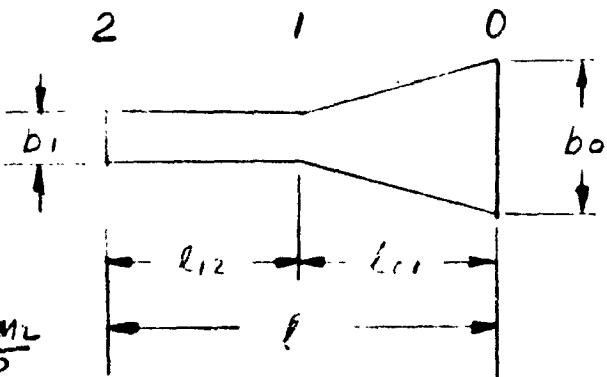


FIG 3

ALSO

$$-\frac{b_1}{M_2} = \frac{b_0}{P_L + M_2} \quad \text{OR} \quad 1 + \frac{-b_1/M_2}{P_L + M_2}$$

$$C = \frac{b_0 - a_1}{b_0} = \frac{b_0 - \left(\frac{b_0 M_2}{P_L + M_2}\right)}{b_0} \quad 1 + \frac{M_2}{P_L + M_2} = \frac{P_L + 2M_2}{P_L + M_2}$$

$$1 - C = 1 - \frac{P_L + 2M_2}{P_L + M_2} = \frac{-M_2}{P_L + M_2}$$

$$I_1 = I_0(1-C) = -\frac{M_2 I_0}{P_L + M_2}$$

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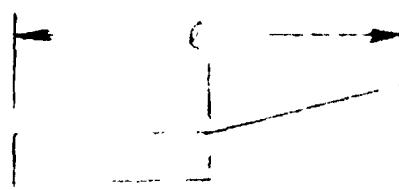
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TRI-FLEX PIVOT ANALYSIS
WITH VARIABLE WIDTH SPRINGS

JOB 60008

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NOTE $M_1 = -M_2 > 0$
(THUS M_1 IS IN THE SAME
DIRECTION AS P)

ANGULAR DEFLECTIONS

$$\theta_{M_1} = -\frac{M_1 \cdot I_{c1}}{E J_c} \ln \left(\frac{1-c}{c} \right) *$$

$$= -\frac{(-M_2)}{E J_c} \frac{\left(\frac{P_L + 2M_2}{P} \right) \ln \left(\frac{-M_2}{P_L + M_2} \right)}{\frac{P_L + 2M_2}{P_L + M_2}}$$

$$\theta_{M_1} = \frac{M_2 (P_L + M_2) \cdot \ln \left(\frac{-M_2}{P_L + M_2} \right)}{E J_c P}$$

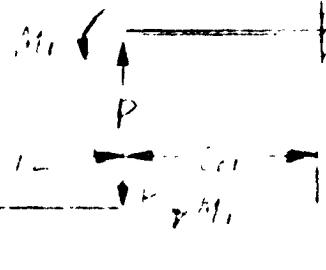


FIG. 4

$$\theta_{P_1} = \frac{P L^2}{E J_c} \left(\frac{1}{c} \right) \left[1 + \left(\frac{1-c}{c} \right) \ln \left(\frac{1-c}{c} \right) \right]$$

FPCM FEB. 17, 1955 C COPY
PRODUCT FNAL BY DR K. MAIER

$$= \frac{P \left(\frac{P_L + 2M_2}{P} \right)^2}{E J_c \left(\frac{P_L + 2M_2}{P_L + M_2} \right)} \left[1 + \frac{\frac{-M_2}{P_L + M_2}}{\frac{P_L + 2M_2}{P_L + M_2}} \ln \left(\frac{-M_2}{P_L + M_2} \right) \right]$$

$$\theta_{P_1} = \frac{(P_L + M_2)(P_L + 2M_2)}{E J_c P} \left[1 - \left(\frac{M_2}{P_L + 2M_2} \right) \ln \left(\frac{-M_2}{P_L + M_2} \right) \right]$$

$$\theta_{M_2} = \frac{M_2 i_{c2}}{E J_c} = \frac{M_2 \left(-\frac{2M_2}{P} \right)}{E \left(\frac{-M_2 I_c}{P_L + M_2} \right)} = \frac{2M_2 (P_L + M_2)}{E J_c P}$$

$$\theta_{P_2} = \frac{P (i_{c2})^2}{E J_c} = \frac{P \left(-\frac{2M_2}{P} \right)^2}{E \left(\frac{-M_2 I_c}{P_L + M_2} \right)} = -\frac{2M_2 (P_L + M_2)}{E J_c P}$$

$$\theta_{M_2} + \theta_{P_2} = 0$$

$$\therefore \theta_1 = \theta_2 = \theta_{M_1} + \theta_{P_1}$$

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TRI-FLEX PIVOT ANALYSIS
WITH VARIABLE WIDTH SPRINGS

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LATERAL DEFLECTIONS

$$\delta_{M_1} = \frac{M_1 (I_{c1})^2}{EI_c} \left[\frac{(1-c) \ln(1-c) + c}{c^2} \right] \quad \text{SEE APPENDIX PAGE 8 FOR DERIVATION}$$

$$= \frac{(-M_1) \left(\frac{P_c + 2M_2}{P} \right)^2}{EI_c} \left[\frac{\left(\frac{-M_2}{P_c + M_2} \right) \ln \left(\frac{-M_2}{P_c + M_2} \right) + \frac{P_c + 2M_2}{P_c + M_2}}{\left(\frac{P_c + 2M_2}{P_c - M_2} \right)^2} \right]$$

$$\delta_{M_1} = \frac{M_1 (P_c + M_2)}{EI_c P^2} \ln \left(\frac{-M_2}{P_c + M_2} \right) - \frac{M_1 (P_c + M_2) (P_c + 2M_2)}{EI_c P^2}$$

$$\delta_{P_1} = \frac{P l_{c1}^3}{EI_c} \left(\frac{1}{2c} \right) \left[1 - \frac{2(1-c)}{c^2} \left\{ (1-c) \ln(1-c) + c \right\} \right] \quad \begin{matrix} \text{FROM FEB. 17, 1959} \\ \text{PRIV. ENGINEERING} \\ \text{BY DR. K. MAIFELD} \end{matrix}$$

$$= \frac{P \left(\frac{P_c + 2M_2}{P} \right)^3}{EI_c \cdot 2 \left(\frac{P_c + 2M_2}{P_c + M_2} \right)} \left[1 - \frac{2 \left(\frac{-M_2}{P_c + M_2} \right)}{\left(\frac{P_c + 2M_2}{P_c + M_2} \right)^2} \left\{ \left(\frac{-M_2}{P_c + M_2} \right) \ln \left(\frac{-M_2}{P_c + M_2} \right) + \frac{P_c + 2M_2}{P_c + M_2} \right\} \right]$$

$$= \frac{(P_c + 2M_2)^2 (P_c + M_2)}{2EI_c P^2} \left[1 - \frac{2M_2^2}{(P_c + M_2)^2} \ln \left(\frac{-M_2}{P_c + M_2} \right) + \frac{2M_2}{(P_c + 2M_2)} \right]$$

$$\delta_{P_1} = \frac{(P_c + M_2)(P_c + M_2)(P_c + 4M_2)}{2EI_c P^2} - \frac{M_2^2 (P_c + M_2)}{EI_c P^2} \ln \left(\frac{-M_2}{P_c + M_2} \right)$$

$$\delta_{M_2} = \frac{M_2 l_{c2}^2}{EI_c} = \frac{M_2 \left(-\frac{2M_2}{P} \right)^2}{2E \left(\frac{-M_2 I_c}{P_c + M_2} \right)} = -\frac{2M_2^2 (P_c + M_2)}{EI_c P^2}$$

$$\delta_{P_2} = \frac{P (l_{c2})^3}{3EI_c} = \frac{P \left(-\frac{2M_2}{P} \right)^3}{3E \left(\frac{-M_2 I_c}{P_c + M_2} \right)} = \frac{8M_2^2 (P_c + M_2)}{3EI_c P^2}$$

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TRI-FLEX PIVOT ANALYSIS
WITH VARIABLE WIDTH SPRINGS

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FROM PAGE 2

$$\theta_1 = \theta_2 = \theta_{M1+M2} = \frac{M_2(P_e + M_2)}{EI_c P} \ln \left(\frac{-M_2}{P_e + M_2} \right) + \frac{(P_e + M_2)(P_e + 2M_2)}{EI_c P} \left[1 - \left(\frac{M_2}{P_e + M_2} \right)^2 \right] \frac{-M_2}{P_e + M_2}$$

$$\theta_1 = \theta_2 = \frac{(P_e + M_2)(P_e + 2M_2)}{EI_c P} \quad (1)$$

FROM PAGE 3

$$\begin{aligned} J_2 &= i_{M1+M2} + i_{M2} + i_{P_e} + l_{12} \theta_1 \\ &= \left[\frac{M_2^2 (P_e + M_2)}{EI_c P} \ln \left(\frac{-M_2}{P_e + M_2} \right) - M_2 (P_e + M_2) \frac{(1 + 2M_2)}{EI_c P} \right] \\ &\quad + \left[\frac{(1 + M_2)(1 + 2M_2) + 19M_2^2}{2EI_c P} - \frac{M_2^2 (P_e + M_2)}{EI_c P} \ln \left(\frac{-M_2}{P_e + M_2} \right) \right] \\ &\quad + \left[-\frac{2M_2^2 (1 + M_2)}{EI_c P} \right] + \left[\frac{5M_2^2 (1 + M_2)}{2EI_c P} \right] + \left(-\frac{2M_2}{P_e} \right) \frac{(P_e + M_2)(P_e + 2M_2)}{EI_c P} \\ &= \left(\frac{P_e + M_2}{EI_c P} \right)^2 \left[(P_e + 2M_2)(-M_2 + \frac{P_e + 9M_2}{2} - 2M_2) + (-2M_2^2 + \frac{5}{2}M_2^2) \right] \end{aligned}$$

$$J_2 = \frac{(P_e + M_2)}{EI_c P} \left[(P_e + 2M_2) \cdot \frac{P_e}{2} \cdot M_2 + \frac{2}{3} M_2^3 \right] \quad (2)$$

FROM FIG 5

$$S_2 = \theta_2 (\ell - R)$$

$$\text{SETTING } K = \frac{R}{2}$$

$$J_2 = \theta_2 \ell (1 - K)$$

$$\text{CR } K = \frac{\theta_2 \ell - S_2}{\theta_2 \ell}$$

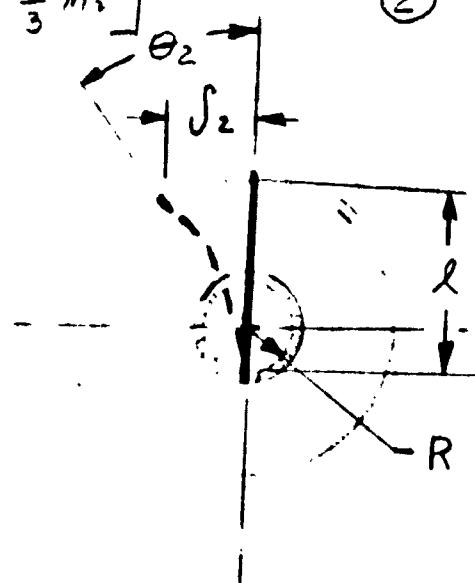


FIG. 5

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WITH VARIABLE WIDTH SPRINGSJOB 6100DATE 3 May 66

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$$\begin{aligned}
 K &= \frac{\theta_2 L - f_2}{G_2 L} = \frac{(P_c + M)(F_c + 2M_1)I^2}{EI_c P} - \frac{(P_c + M)F_{c2} + 2M_1 \left(\frac{P_c}{2} - M_1 \right) + \frac{2M_1^2}{3}}{EI_c P} \\
 &= \frac{P_c + M_1 (P_c + 2M_1) I^2}{EI_c P} \\
 &= \frac{(P_c + 2M_1)P_c - (P_c + 2M_1)\left(\frac{P_c}{2} - M_1\right) - \frac{2M_1^2}{3}}{(P_c + 2M_1)P_c} \\
 &= \frac{P_c^2 + 2P_c M_1 - \frac{P_c^2}{2} + 2M_1^2 - \frac{2M_1^2}{3}}{(P_c + 2M_1)P_c} \\
 K &= \frac{3P_c^2 + 12P_c M_1 + 5M_1^2}{6P_c (P_c + 2M_1)} \quad (3)
 \end{aligned}$$

FROM FIG. 5 ON 1 LEAF OF TRI-FLEX PIVOT

$$\text{TORQUE } T = M_2 + P(2-R) = \left(\frac{M_2}{P_c} + 1-K\right)P_c$$

$$\text{ON FULL PIVOT } T = 3\left(\frac{M_2}{P_c} + 1-K\right)P_c.$$

$$P_c = \frac{T}{3\left(\frac{M_2}{P_c} + 1-K\right)} \quad (4)$$

$$\text{ON 1 LEAF: } M_2 = P_c + M_1 = \frac{P_c + M_1}{K} = \frac{T}{3\left(\frac{M_2}{P_c} + 1-K\right)} \quad (5)$$

DIVIDING Eqs (4) & (5): $\frac{P_c}{M_2} = \frac{P_c + M_1}{P_c}$

$$\frac{1}{K} = \frac{3\left(\frac{M_2}{P_c} + 1-K\right)P_c}{(P_c + M_1)(1 + \frac{3M_1}{P_c})} = \frac{3EI_c I^2 \left(\frac{M_2}{P_c} + 1-K\right)P_c}{(P_c + M_1)(1 + \frac{3M_1}{P_c})} \quad (6)$$

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TRI-FLEX PIVOT ANALYSIS
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$$SECTION \quad \frac{M_2}{P_E} = A$$

EG ⑤ DEFLECTIONS:

$$K = \frac{3 + 12A + EA^2}{6(1+2A)} \quad (7)$$

EG ⑥ DEFLECTIONS:

$$\frac{I}{\theta} = \frac{\frac{3EI}{l} (A + 1 - k)}{(1+A)(1+2A)} = \frac{EI_o}{l} B \quad (8)$$

WHERE B IS THE SPRINGS RATE COEFFICIENT

EG ⑦ BENDING MOMENTS:

$$M_1 = \frac{(1+A) T}{3(A+1-k)} \quad (9)$$

THEN Z_0 (SECTION MODULUS AT SECTION O)

$$Z_0 = \frac{M_1}{S_{ALL}}$$

$$I_o = Z_0 \frac{l}{3} \quad \text{WHERE } l = \text{SPRING THICKNESS}$$

$$l = \frac{EI_o B}{\frac{T}{\theta}}$$

INNER DIA. OF OUTER RING

$$D = 2(l - r) = 2l(1 - k) = 2(1 - r) \frac{ET_o B}{\frac{T}{\theta}}$$

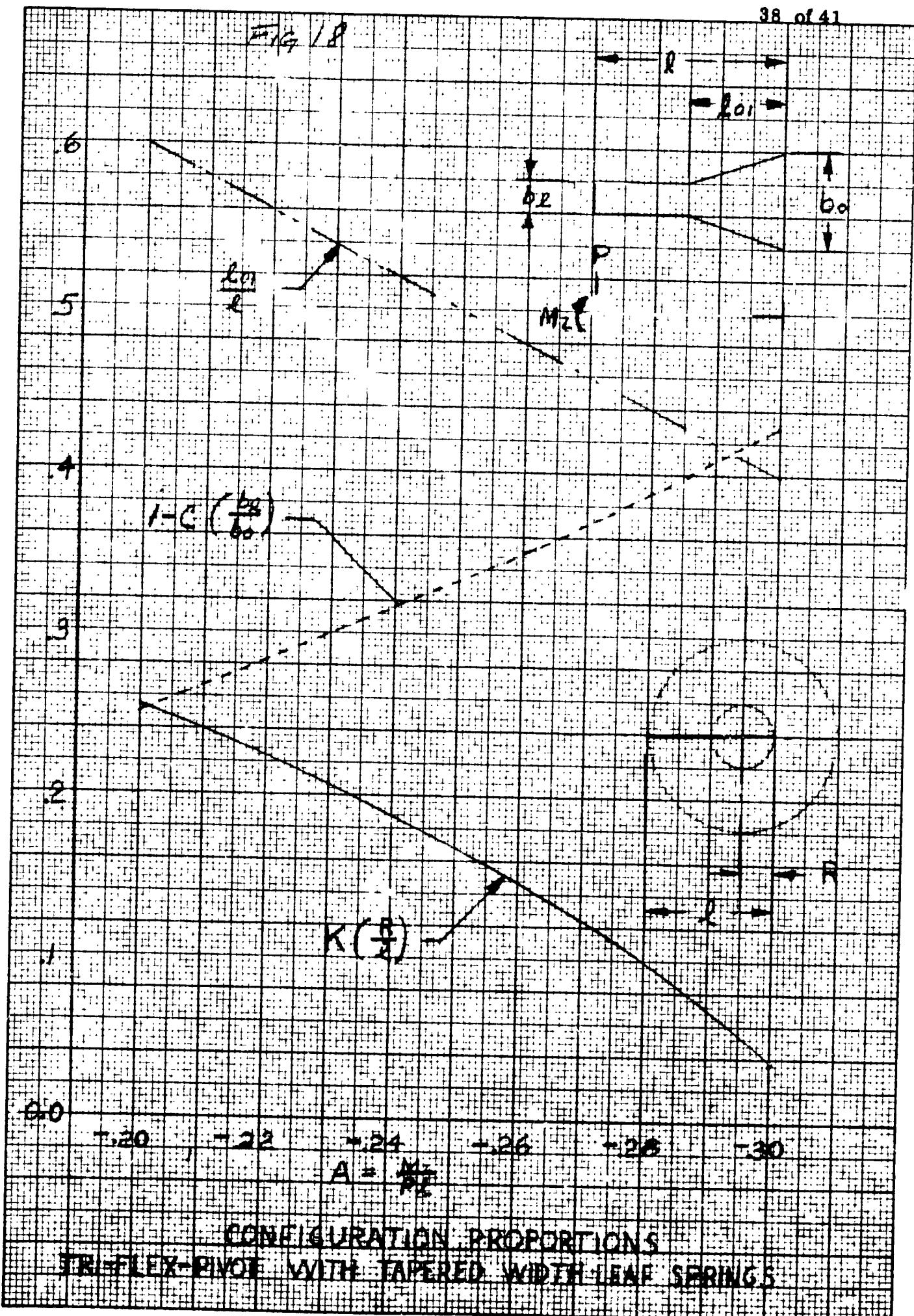


FIG 19

SPRING RATE COEFFICIENT B

$$B = \frac{(3 + 12A + 6A^2)(3 + 6A + 4A^2)}{12(1 - A)(1 + 2A)^2}$$

-20 -22 -24 -26 -28 -30

A = M

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VARIABLE WIDTH CANTILEVER
 WITH END MOMENT

THE MOMENT OF INERTIA OF
 THE CANTILEVER BEAM SHOWN IS

$$I = I_c - \left(\frac{I_c - I_e}{\frac{b}{l}} \right) x$$

$$I = I_c \left[1 - \left(\frac{b_c - b_e}{b_e} \right) \frac{x}{l} \right]$$

$$\text{SETTING } C = \frac{b_c - b_e}{b_e}, \text{ AND } x = l$$

THE DEFLECTION PROFILE IS TO

$$I = I_c (1 - Cr) \quad (1)$$

THE DEFLECTION FORMULA OF THE LOAD
 CURVE IS

$$\frac{dy}{dx} = \frac{M}{EI}$$

WHERE M IS THE BINDING MOMENT (CONSTANT)

E IS YOUNG'S MODULUS

I IS THE MOMENT OF INERTIA (Eq. 1)

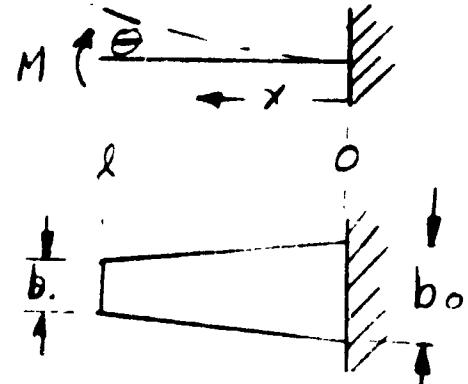
y IS THE DEFLECTION AT ANY POSITION

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI} dx = \int \frac{M}{EI_c (1 - Cr)} dx$$

$$= \frac{M}{EI_c} \int_0^x \frac{dx}{1 - Cr} = \frac{M}{EI_c} \left[\frac{1}{C} \ln(1 - Cr) \right]_0^x \quad (2)$$

$$= \frac{M}{EI_c} \left\{ \left[-\frac{1}{C} \ln(1 - C) \right] - \left[-\frac{1}{C} \ln(1 - 0) \right] \right\}$$

$$\text{AT } x = l \quad \theta = -\frac{M}{EI_c} \left[\frac{1}{C} \ln(1 - C) \right] \quad (3)$$



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VARIABLE WIDTH CANTILEVER WITH END PLATE

$$\frac{dy}{dx} \in - \frac{M}{EI} \left[-\frac{1}{c} \ln(1-c) \right]$$

$$y = \int_{EJ}^M \left[-\frac{1}{c} \ln(1-c) \right] dx$$

$$= -\frac{M}{EI_c} \int_0^x \ln(1-c) dx$$

$$= -\frac{M}{EI_c} \left[\frac{(1-c)x - (1-c) - (1-c)}{c} \right]^x_0$$

$$= -\frac{M}{EI_c} \left[\frac{(1-c)x - (1-c) - (1-c)}{c} - \frac{1 \ln(1) - (1-c)}{c} \right]$$

$$y = -\frac{M}{EI_c} \left(\frac{1}{c^2} \right) \left[(1-c)x \ln(1-c) + c \right]$$

$$\text{AT } x = 0 \quad y = -\frac{M}{EI_c} \left(\frac{1}{c^2} \right) \left[(1-c) \ln(1-c) + c \right]$$